Taking Away the Fear of Solving Mathematical Problems

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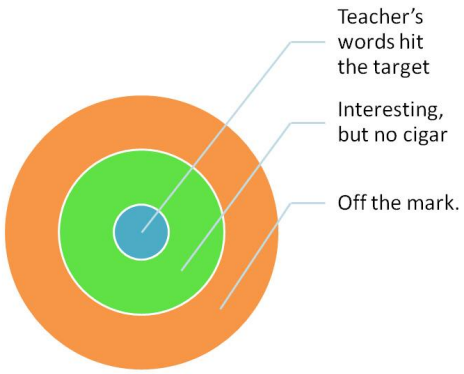
ABSTRACT

Taking away the fear of solving mathematical problems is the central theme of this review paper and to achieve this goal, teachers should sense the need to engage their students by whatever means possible. The studies which have been reviewed in this paper, imply that if a learner is actually interested in the topic being studied – or perhaps he or she simply has a natural ability in solving that particular type of problem – then, they are more likely to succeed and less likely to develop fear and anxiety.

This paper will first review the role curiosity plays in the affectivity of problem solving (Goldin, 2000) and then will consider the part that the affects of interest and excitement play in getting started solving mathematical problems (Ormrod, 2008). It will then examine a the set of cognitive abilities that makeup the controversial theory of Multiple Intelligences (Gardner, 1983) and then it will review the use of Multiple Representations, which illustrate four different approaches to teaching the **same** mathematical concept (Jones & Swan, 2006). This paper will report on how Multiple Strategies are used to present a **single** problem, solved with five different techniques (Meyer, 1999). Finally, this paper will reveal how the Credit Card project changed moans to laughter and enthusiasm (Marks, 2000). Throughout all of these theories, ideas, and experiments, consider these two fundamental questions:

1. Can mathematical problem solving be made more "user-friendly" in order to increase the interest level for the new learner?
2. Once learners are interested and get engaged in solving mathematical problems, can we keep that interest after the "shine wears off" -- when basic numerical processing and logical deduction replaces basket ball & credit card games?

TAKING AWAY THE FEAR OF SOLVING MATHEMATICAL PROBLEMS



Mathematical problem solving begins with curiosity and ends with either satisfaction or with anxiety, frustration, and fear (Goldin, 2000). Curiosity can be generated internally by the learner or externally by teachers. How can teachers raise the interest level and at the same time raise the learner's curiosity? Jeanne Ormrod said that learners that have interest in a particular topic or activity are learners that find it intriguing and engaging. In order to relate 'interest' to 'memory retention', students were asked to prioritize six different magazines from their most favorite to least favorite (i.e. *Better Homes and Gardens, National Geographic, Newsweek, People, Popular Mechanics, and Sports Illustrated*), they also recorded how much information they thought they would remember after reading those magazines. Their obvious response was that they will retain more long-term memory data about magazines that were at the top of their list, as compared to no information from their least interesting magazines (Ormrod, 2008). This paper will review studies that address internally generated curiosity – curiosity generated by the student's inherent abilities and skills; or as a result of their hobbies and interest in certain topics, activities, or objects; or as a result of their naturally occurring intelligence (Gardner, 1983). Gardner's natural intelligence categories have opened doors to 7 exceptional cognitive abilities: verbal-linguistic, logical-mathematical, musical, visual-spatial, bodily kinesthetic, interpersonal and interpersonal intelligences.

MULTIPLE INTELLIGENCES

In *Frames of Mind - the Theory of Multiple Intelligences,* Dr. Howard Gardner referred to Franz Joseph Gall as the creator of phrenology (later 18th century), where the shapes of heads & brains were used to foretell of one's intelligence. Gall and others tried unsuccessfully to extend that concept to traits, characteristics, capabilities, and skills; but what he did succeed in doing was to plant the idea seed, that different parts of the brain may "mediate different functions". While British educational psychologist Charles Spearman spoke of a central processing system in the brain and that one could measure the intelligence thereof; an American psychometrician named L.L. Thurstone believed in independent mental functions of the brain that could be measured by different tasks. Thurstone suggested that such factors as verbal comprehension, word fluency, spatial visualization, associative memory, perceptual speed, and reasoning were a basic set of functions. Howard Gardner harvested those seeds of multiple functions by first examining exceptional brains; the brains of autistic savants and people with brain lesions due to accidents and medical operations. He discovered that when only one portion of a brain is functioning, it may produce a brilliant genius of one form or another.

Gardner's point was that human cognitive abilities are varied and powerful. To put it simply – some people can do better and learn faster at certain things, than those with a higher "I.Q.". Gardner saw extraordinary Linguistic Intelligence in the genius of the word selection of poet's and the creative masterpieces of authors throughout time – convinced that a human's verbal cognitive ability could be measured. He marveled at the Musical Intelligence of adolescents singing every note and word from an opera, after only one sitting – believing that this type intelligence could in fact,

be measured. The Logical-Mathematical Intelligence of gifted individuals is universally acknowledged by most educators. The mental imagery of individuals with Spatial Intelligence is recorded throughout history by designers, architects, and artists. Even simple acts, like typing,

archery, athletics, and playing complex instruments, demonstrate Bodily-Kinesthetic Intelligence with a highly coordinated interplay of brain cells and muscle coordination. Finally, Gardner looked to psychiatrists for the Personal Intelligences – Freud for the introspective type personality (which he later coined Interpersonal Intelligence) and James for the external looking type personality that had an uncanny understanding of why others do what they do (which he later coined Intrapersonal Intelligence). Gardner increased his original list of 7, by adding the Naturalist Intelligence for individuals that easily categorize nature, such as managing endangered species and the categorization of genes, e.g. the genome project. This review does not endorse Gardner's theory, but it does acknowledge that people (students) may be just a little more than a numbered intelligence quotient and that with over a hundred years of scientific research -- the intricate human brain is still not fully understood. Further, naturally occurring skills and abilities are sure to have some level of cognitive application, such as reframing a concept to be communicated in terms of music problems for some and in terms of sports problems for others. One might question how that can be done. Stay tuned, because the next section is about Multiple Representations.

Multiple Intelligences

-Verbal-Linguistic
reading, listening, discussing

-Logical-Mathematical
solving problems/puzzles

-Musical
singing, dancing, playing music

-Visual-Spatial
solving mazes, graphics, art

-Bodily-Kinesthetic
manipulatives, sports, games

-Intrapersonal
journal-writing, imagination

-Interpersonal
group projects, role playing

MULTIPLE REPRESENTATIONS

If a child has inherited a talent for music, combined with an auditory modality, can't an educator design an activity that will address that talent? Teaching an algebra lesson in 4 different representations, each addressing the student's Multiple Intelligence is accomplished by Dr. Doug Jones and Dr. Gerry Swan at Appalachian State University (Jones & Swan, 2006). Multiple Representations, demonstrated in Activities #1 through #4, plus an added activity using verbal skills, that give examples of teaching to and through each of the intelligences.

The lesson being taught was part of an algebra class, which taught the students about the mechanics of solving systems of equations. Activity #1 was a typical math word problem – the Smith family left the campground at 8:45 a.m. and started on a trip of 137 miles toward their home at 40 mph, but their neighbors were delayed leaving the campground by 30 minutes and they travelled at 55 mph; when and where do the two cars meet? The students made a table and plotted the trips on a graph, the same as math students do all over the world. Students with natural mathematical abilities and a logical mind gravitated toward this activity.

Multiple Representations

ACTIVITY #0
- Verbal
- Linguistic

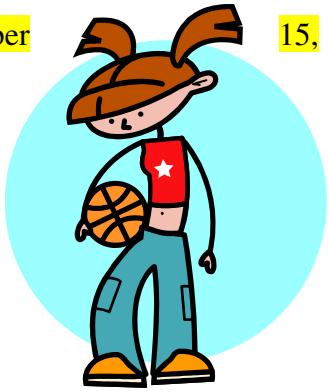
ACTIVITY #1
- Mathematical
- Logical

ACTIVITY #2
- Kinesthetic
- Spatial

ACTIVITY #3
- Musical
- Auditory

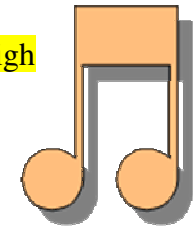
ACTIVITY #4
- Visual
- Intrapersonal

Activity #2 was a video of a basketball game and the 'Eagles' got the ball and cleared the ball to the side of the court to start a fast break with number 15, Derek Moss driving toward the opponent's basket. The opposing point-guard, number 4, Rajon Canon, took off a half second later to try and catch Moss and block the shot. The court is 92 feet long.



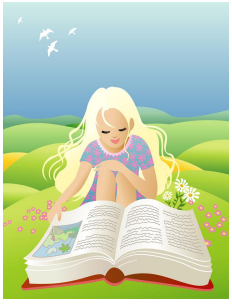
How much faster than Moss will Canon have to run, if he is going to catch up? Students whose learning depends upon kinesthetic and visual skills, or learners who are strong in spatial intelligence, were excited to get started with this activity.

Activity #3 has to do with students with a strong in music intelligence and it consists of two xylophone players, one starting at the low-notes and the other starting at the high notes. If both approach one another one note at a time, through two octaves and in the same meter, will they play the same note at the same time? Using graphs of linear functions, the learners solved this problem in basically the same manner as the first two activities, but with music.



Activity #4 has to do with mixing paint colors together to achieve the same exact color as a given color. Again the paint increments are plotted and an optimum point is achieved when the two functions meet and the system of equations is solved. This approach appealed to those students with an acute visual modality, plus those with an intrapersonal intelligence and a sense of when to stop pouring paint -- so that the perfect paint blending-point would not be missed.

Activity #0 was added just to demonstrate that if one wanted to extend this exercise, one could ask the verbally gifted to describe the mechanics of the system of equations with a linguistic flair; perhaps with historical factoids and practical descriptions. This math lesson was accomplished by using Multiple Representations and was enjoyed by most of the learners – all of which learned exactly the same math lesson.



How about using the same problem and allowing the students to decide which strategy is best for them? Next up for review is the traditional application of Multiple Strategies.

MULTIPLE STRATEGIES

In Multiple Strategies = Multiple Challenges, Meyer demonstrated that different strategies can be applied to a single mathematics problem – reinforcing the concept that teachers can teach the same topic using a variety of cognitive strategies (Meyer, 1999). Multiple Strategies such as: draw a picture, use a model, make a list, eliminate possibilities, use symmetry, solve an equation, look for a pattern, work backward, and guess & check; are a set of traditional techniques or strategies that are used successfully to solve mathematical problems. Each of these strategies are rooted in the many of the natural abilities of the human brain, with 'solving an algebraic equation' being demonstrated, along with algebraic thinking being is used successfully in Strategy #1. The elimination of bad answers through a guessing scheme in Strategy #2 seems to more natural to certain students, as can be detected in the student saying "I pretended that the umbrella and a cap cost the same". Some personality types have a problem with 'pretending' (especially in the logical domain of mathematics). Strategy #3 was adopted by an individual who was number smart – being able to scan columns and compare the various summations, while analyzing the probable correct answer at a very high level.

Strategy #4 was clearly a job for a spatial analysis expert, without the need for guessing or the use of algebraic equations. Strategy #5 utilizes a numbers of skills and talents by making a list, visually identifying patterns, and drawing conclusions.

Multiple Strategies

STRATEGY #1
- Solve an Equation
- Algebraic Thinking

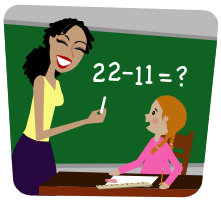
STRATEGY #2
- Guess & Check
- Eliminate Possibilities

STRATEGY #3
- Working Backward
- Look for a Pattern

STRATEGY #4
- Draw a Picture
- Spatial Talents

STRATEGY #5
- Make List / Chart
- Visual Talents

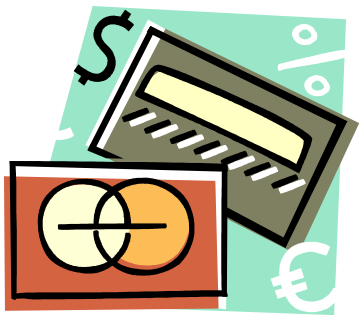
Meyer discussed the challenges to the teacher in creating a classroom environment that can successfully implement Multiple Strategies, including opening up class discussion for students to explain their solution strategy, where the classroom becomes a public forum and the strategies that are presented are open to criticism and praise (Meyer, 1999, p. 522). The teacher must also recognize (and appreciate) the various mathematical understanding that is revealed by the various strategies – differing in abstraction, in efficiency, and in extent of applicability. One big plus of this open-solution approach is that a teacher can now easily detect those that continue in an inefficient or unsophisticated fashion and thereby identify a need for special attention to that particular child. The teacher must also be aware of social changes as a result of the open atmosphere and be able to adjust that situation as required – where the "smart kids" will soon gain attention and become informal leaders. The last aspect of this open environment is considered a negative -- when teachers do not know the answer. It is okay, because honesty and openness is actually appreciated in listening to new approaches, especially by the students.

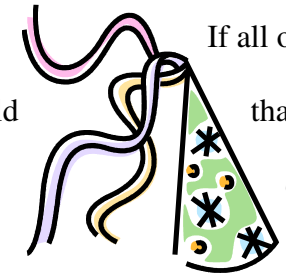


How about letting the students have fun, while they are exercising their natural abilities and showing interest to items in a gift catalog, spending a large amount of money.

CREDIT CARD QUIZ

In Jeff Marks' article entitled "*Credit Card Quiz"*, students began the class period with an announcement of a dreaded pop quiz. A few moans were uttered as the students took out their pencils and paper. The quiz was revealed to the students, and the moans gave way to excited chatter and laughter. So began the pre-algebra class's journey into the exciting world of buying their favorite items and getting to use credit cards in so doing. The



challenge for the teacher was to meld the content from the textbook with real-life application by using teaching strategies, based on Multiple Intelligences (Marks, 2000). The teacher needed to focus on the goal of teaching to and through each of the intelligences when practical, but not to stretch the content of the curriculum just to reach other intelligences. The students were first asked to match slogans with credit card brand names – like "It's everywhere you want to be…" and "There are some things money can't buy…" etc. and the kids loved it. They were also asked about media advertisements they had seen on TV, magazines, or the internet and to analyze the effectiveness of the advertising campaign. They began the Credit Card project by designing their own credit cards – size, color, and creative logo and then designed their own advertising campaign – they presented their ads to the class, creative video presentations and brightly colored posters for their new logos.  If all of that didn't excite the students, they really got engaged when they were told that tomorrow they would be given $10,000 to spend on items of their choice from a shopping catalog. The next day they bought "a bunch of cool stuff" from gift catalogs. Some bought clothes, others were very happy to purchase an HDTV for $3,000 – to the applause of their classmates! While music blared from a radio – the students went on a 'spending-spree' and at the end of class, their purchases were turned-in and recorded.

The next day each student was greeted at the door with a bill for all credit card charges! Gloomy music played on the radio and the "Grim Reaper" was displayed at the front of the room. Nervous kids wandered what was going on, until they were told that the first payment on their credit card debt was due today – but not to worry, the students only had to pay $50/month with an interest rate of 15%. The students liked that option because $50 was not all that much money. Upon calculating what they

would be paying, some three years and others 15 years into the future, they began to realize what charging money on a credit card was all about. They were asked to compute monthly and daily interest rates and analyze what the low payment and the rather high interest rate was going to do to their finances over time. Many students got disgusted and exclaimed they would never be able to pay-off the debt. They tried increasing the monthly payments while one student commented, "I would have been better off to save up the money and buy it for cash".

This kind of exposure to real-world experiences and the ability to be creative in the design of your own advertising campaign; your own credit card logo and ad presentation in front of the class; the experience of buying items that fit into your personal interests and abilities; all of this was directed at the set of multiple intelligences that is present in all of us. The Credit Card project was not elegant and maybe didn't really connect with some of the special intelligences, but it certainly got the students engaged and was a fascinating way to teach life lessons about borrowing money. The credit billing calculations were so much a part of the exercise they may not have realized they were doing mathematics; by learning how to compute monthly interest charges, optimum monthly payments, using annual and monthly interest rates, these learners will be storing this lesson in their long-term memory for future purposes.

CONCLUSION

This review has explored a variety of techniques that generate interest and curiosity in students toward solving mathematical problems. Many of these approaches are time-consuming for the teacher and even unrealistic, considering the pressures of the modern curriculum and administrative duties. One cannot be so naïve as to think our public classrooms of today are fun, flashy, exciting places to be, with extra time to create 'multiple anythings' – but by the same

token, educators cannot be resigned to 'business as usual'. There needs to be a movement toward an excellence in education that does not leave out the intellectual strengths of our learners, in favor of the one-size-fits-all method. Teachers need to talk this subject up – they need to probe and plan and even push a bit – to expand horizons and question tradition. Should not each capable student feel good about mathematics and not be afraid to be challenged?

As far as Gardner's theory of Multiple Intelligences, not everyone buys-into his theory; but there cannot be disagreement that our learners are multifaceted, filled with a wide array interests, skills, talents, abilities, likes, and dislikes. That is reason enough to learn more about this topic and attempt to reach out and touch their areas of interest. Educators should be able to identify a student's natural abilities, hobbies, and interests in order to develop their curiosity and to engage the student in finding solutions to mathematical problems. As child after child, becomes intrigued with math, teachers will have succeeded in taking away some of the fear.

Can mathematical problem solving be made more "user-friendly" in order to increase the interest level for the new learner? Yes it can be and yes it should be. Once learners are interested and get engaged in solving mathematical problems, can we keep that interest after the "shine wears off"? The best answer to this question is that the shine never really has to wear off – the interest and capability levels need to be nurtured and fine-tuned as long as it is necessary, to keep the students engaged. In time, the mature problem solver will let you know that it is no longer necessary to make mathematics "user-friendly" and that the student is now in control – and is no longer afraid.

REFERENCES

Baroody, A.J. (1993). Problem Solving, reasoning, and communicating, K-8: Helping children think mathematically. New York: MacMillan.

Gardner, H. (1983). Frames of Mind: The Theory of Multiple Intelligences. New York: Basic Books.

Goldin, G. A. (2000). Affective Pathways and Representation in Mathematical Problem Solving, Mathematical Thinking and Learning, 2:3, pp. 209-219.

Jones, D., & Swan, G. (2006). Systems of Equations, Representations, and Habits of Mind. ON-Math 2006-2007, 5(1). Retrieved from http://www.nctm.org/publications/content.aspx?id=8598: An electronic publication of the National Council of Teachers of Mathematics (NCTM).

Marks, J. (2000). Credit Card Quiz. Teacher to Teacher, 6(3), 150-154. Retrieved from http://www.nctm.org: An electronic publication of the National Council of Teachers of Mathematics (NCTM)

Meyer, M.R. (1999). In S.N. Friel (Ed.), Reflections on Practice - Mathematics Teaching in the Middle School: Vol. 4. Multiple Strategies = Multiple Challenges (8, pp. 519-523). Chapel Hill, NC: University of North Carolina, School of Education.

Ormrod, J.E. (2008). Educational Psychology, Developing Learners, 6th Edition. New Jersey: Carlisle Publishing Services.